The Royal and Ancient Golf Club of St Andrews

and

United States Golf Association

Technical Description of the Pendulum Test

Revised Version

Discussion of points raised during Notice & Comment period

(November 2003)

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1 Introduction

In this document we give details of the technical aspects of the pendulum. Most of these are in response to questions which have been raised during the Notice and Comment process. This report is intended to provide information of a technical nature, but we shall start by summarising the salient points. We note that the report is divided into sections: technical details of the science behind the test; technical details of the resolutions of questions raised concerning effects on the test and finally, discussion of the test and submission protocol.

Before we start our description we mention that at points in this report we shall highlight the fact that our intent is to determine the conformance status of the club and as such we concentrate on the range of clubs whose performance is close to the limit. Although we note that the pendulum does produce a good indication of the performance of clubs away from this limit we do not intend to unduly compromise our ability to judge conformance by the consideration of these clubs. In summary we intend to ensure that the smallest errors in the judgement of characteristic time occur in the neighbourhood of the limit, although obviously we must ensure that correct judgements are also made for clubs removed from the limit.

1.1 Technical details of the science behind the test

In this section we discuss the algorithm and the model used in the test, including the statistical methods which are used to evaluate each individual test. We also give details of a Gage R & R analysis which deals with the repeatability and reproducibility associated with two aspects of the test, namely the marking of the centre location on the face and the variability associated with machines and operators. In response to comments we have revisited the choice of exponent in the model and have found that an exponent of $k = 0.33$ more accurately captures the physics of the impact between two curved bodies where one is intrinsically flexible. This has necessitated a change in the associated limit to 239μs. We have further refined the data analysis method to reduce the susceptibility of the technique to noise.

We have also modified the software to include the improved statistical analysis and to extend the time over which the test is run from 500 microseconds to 1000 microseconds. We shall also comment on the provision of a calibration object. We conclude this section with a discussion of the setting of the limit and the correlation between the coefficient of restitution and the characteristic time.

1.2 Technical details of the resolutions of questions raised concerning effects on the test

In this section we comment on the issues raised concerning the effects of club head mass, loft, shafting and coatings on the face. These comments will be based on our scientific findings. In brief the club head mass is found to have an effect, albeit secondary. In essence the characteristic time increases as the club head mass increases. It has been found that the club's loft will not have an effect on the test provided the test is performed carefully in that it must be assured that the correct location is hit on the face. We have shown through our own tests, that the shafting of the club plays no rôle in the test; although we add the slight caveat that the attachment must not add undue mass to the head. In order to test a club with a 'coated' face it is necessary to apply an additional test, namely a measurement of the thickness of this layer. This extra measurement is only applied where the presence of the coating may have influenced the conformance of a club. Note that no such clubs have been submitted to the ruling bodies for conformance evaluation.

1.3 Discussion of the test and submission protocol

We shall include a discussion of the test protocol and the way in which a conformance decision is reached. Details are also given of the way in which clubs should be submitted for conformance measurements.

2 Technical Details of the Test

We shall start this section by describing the underlying method with which the test is applied. This is to facilitate the forthcoming discussion.

- The centre of the club face is identified and marked with a small circle. This actually circumscribes the intended impact location.
- Preferably, the club head shall be shafted. A removable shaft may be used. This is then correctly mounted in the pendulum device; this entails ensuring that the club face is perpendicular to the impact in the heel-toe axis and that the centre of the face is in position (using the laser pointer).
- The club is hit nine times (three times at each velocity setting). For each hit the signal is analysed visually and the acquired characteristic time is checked for gross errors.

- The signal from the accelerometer is filtered and integrated to generate the velocity as a function of time. The rising section of the curve is fitted using a cosine curve. The time taken to rise from 5% to 95% of the maximum is recorded.

- The software analyses the data and checks that none of the points are outliers: If they are, the offending point (or points) are identified and these are retested (and the calculation repeated).

- A line of best fit is applied to the data (using a least squares approximation) of the form
$$T = A + BV^{-k},$$
where V is the velocity derived from the pendulum test and T is the time for the velocity to rise from 5% to 95% of this value (as defined above). The constants in the above expression: A the intercept and B the slope are derived from the nine data points.

- A 99% confidence interval is calculated for the value of A (this is dubbed the infinite velocity characteristic time herein) and this is used in the forthcoming conformance decision. If the confidence interval lies wholly below the limit plus the tolerance then the club is deemed to conform to this test.

- If the upper limit of the confidence interval is above the limit then the club is hit a further nine times and the above procedure is repeated, except this time solely the calculated value of A is used to determine the conformance of the club.

- We add the further point that if the slope, B, exceeds 20 units and certain other conditions are met the club will be tested for a coating on the face (this procedure will be detailed in due course).

We shall not dwell on the technical details of the acquisition of the data and refer you instead to the previous technical report. Basically, the signal is collected from the accelerometer via the signal conditioner and the electronic oscilloscope ADC212/50 from Pico Tech (in fact we have now organized a combined unit ADC212/50(Special Order S167) with a signal conditioner, www.picotech.com). The signal is filtered using a truncated Fourier series (see Appendix A.1) and the filtered signal is integrated to find the velocity. This represents the difference in velocity of the pendulum's mass due to the impact with the club face (within the moving frame of the mass). The rising portion of this curve is fitted using a cosine curve which eliminates a sensitivity to the presence of higher frequency modes within the signal.

Since the last release we have extended the time over which the test is performed from 500μs to 1000μs. This was to accommodate clubs with high characteristic times and those with large values of the slope, such as persimmon clubs. It is worth noting that it is necessary to acquire data for a longer time than the actual characteristic time. It is also necessary to increase the number of points acquired (in fact doubling it) so that the resolution of the data is retained. Since the truncation of the Fourier series is based on the overall period of the data, we also need to retain twice the number of Fourier modes in the filtering process (again see Appendix A.1).

The fidelity of the signal can be checked visually. The raw acceleration signal is shown, as is the filtered version, which should resemble the mean of the unfiltered version. The associated velocity curve should appear to rise from zero to a plateau, very much resembling a hyperbolic tangent function. The acceleration signal should be close to zero at the start and end of the signal. We note that the filtering requires this, else one could potentially encounter Gibbs' phenomenon. For a properly acquired signal this should not occur. If this does arise it is likely that the hit has not been properly recorded.

2.1 Simple checks on the acquired signals and data points

Although there is some variation of velocity by machine and by club, it is instructive to ensure that the velocities are similar to

Setting	Approximate velocity
Highest	1.3
Middle	1.0
Lowest	0.65

This may vary between pendulums, but only slightly. This is another advantage of the use of the model, since the actual velocity is calculated and incorporated in the analysis. It is also worth considering the characteristic times at each setting, each of the three measurements at a particular velocity setting should be similar. For metallic clubs, that is those for which a small slope is expected, one should expect quite similar measurements at all three velocity settings; the characteristic time increasing for the lower settings of the pendulum. We add the caveat that for metallic clubs the ordering may not be strict between the different settings. However, for a club with a higher slope, namely a persimmon club (or one with a coated face), there should be an observable change in the times associated with the different pendulum settings (again the lower settings corresponding to higher times).

The size of the confidence interval provides an accurate reflection of the degree to which the results can be used to determine conformance. The confidence interval will generally be less than 8 microseconds wide, in fact in many cases it will be substantially narrower than this. We note that the inclusion of the consideration of the confidence interval within the analysis allows us confidence in our ability to extrapolate using the data.

2.2 The accuracy of the test

There have been many questions concerning the repeatability and reproducibility of the test. These comments fall into two main categories: variations associated the machine and its operation; and variations associated with the marking of the centre of the face (and its identification). It has been found through a Gage R&R analysis (using 3 machines, 3 operators and 6 clubs) that the gage is 6 microseconds (giving 99% inclusion) and this is associated entirely with operator variability (which actually includes centre marking together with placing the club in the device alongside the operation of the test) rather than inter-machine variation. A similar analysis was run on the way in which the club's centre is found and marked, in isolation. It was found that the gage in this case is 2.1mm, that is it is expected that 99% of all future measurements will be within a radius of 2.1mm of the mean of the acquired measurements. We note that this variation is intrinsic to the previous gage. The variation in terms of characteristic time associated with centre marking will of course vary from club to club, but it is very unlikely that this will lead to any substantial change in the characteristic time as has been demonstrated by the overall gage.

We note that in the marking of the centre of the club face the position of the intended impact could coincide with a face marking (punch marks, grooves or decorative markings). In such instances, the position at which the impact will occur is typically moved up the face, albeit slightly. For larger markings, for instance decorative markings (which can be up to 0.375 inches (9.53mm) in length) it may be necessary to perform the measurement at points around the marking and to use an alternative method for evaluating the conformance of the club. In essence

> If a marking prohibits the testing of the club at the centre of the face: if the marking is less than 4.27mm in vertical or horizontal extent (ie groove or punch mark) then a point will be chosen directly adjacent to the location of the intended impact as close

as practical to the centre; if however the size of the centre marking prohibits this (as for a larger decorative marking up to 9.53mm) then we reserve the right to test the club in an appropriate manner in the neighbourhood of this marking.

2.3 The model used to predict the variation of characteristic time with velocity

We have re-evaluated the model used to determine the variation of the characteristic time with velocity. We are now using an alternative exponent k of 0.33. Whilst it is accurate to use an exponent of $k = 0.2$ for the impact between two curved solid bodies, this can be improved upon where one of the bodies is flexible (as it is in this case). In fact the pendulum can be used to accurately predict a true Hertzian contact and this analysis has been run by the way of a check of the device, see Appendix B.

We appreciate that we are trying to measure the flexibility of the club using the test and we have evaluated the likely variation of the flexibility associated with changes in the Young's modulus. Over the range of Young's modulus associated with steel and titanium (as typically used in the manufacture of golf clubs) an exponent of around 0.33 is pertinent, especially for those clubs whose flexibility corresponds to characteristic times close to the limit. We reiterate the point that our intent is to design a machine which determines the conformance of a club rather than being a performance measure. Further details can be found in Appendix C.

2.4 Setting of the limit and the correlation between COR and CT

We have continued to test clubs on the pendulum and this has allowed us to extend our confidence in the ability of the test to provide information about the performance of clubs, especially in the neighbourhood of the limit. To date in excess of four hundred clubs have been tested. In this report we give details of the 60 clubs which have formed the core of our studies.

In Fig. 1 we show the correlation between characteristic time and coefficient of restitution. In order to set the limit we calculate a line of best fit for the data. This shows that the limit of 0.822 corresponds to 239μs and 0.830 corresponds to 257μs. We should stress that this does not represent a relaxation of the limit proposed in a previous document at 232 and 250 respectively; it is merely a recalculation of the available data using the revised exponent $k = 0.33$.

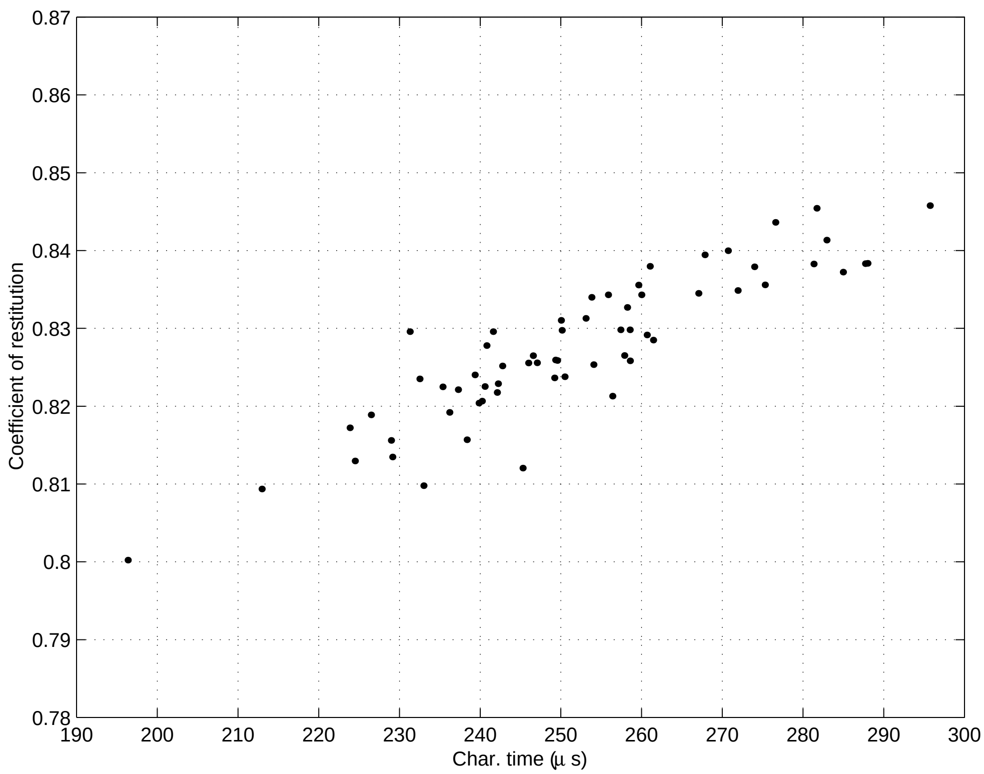


Figure 1: Plot of correlation between characteristic time and coefficient of restitution.

It has been shown in previous documents that the measurement of the infinite velocity characteristic time is predominantly influenced by the flexibility of the club head. Therefore, it would be expected that the coefficient of restitution and the infinite velocity characteristic time may be highly correlated. To test this, a sample of 60 club heads was compiled and measured via both procedures. Figure 1 shows the results of this comparison.

The best fit linear equation correlating the two quantities is:

$$e = 0.718 + 0.000436A,$$

where A is the infinite velocity characteristic time in microseconds.

To maintain consistency with the previous standard, the following limit and tolerance is proposed:

	Current Test	Proposed Test
Test Limit	0.822	239
Tolerance	0.008	18
Limit + Tolerance	0.830	257

We note that after January 1st January, 2004, conformance of a club to spring-like effect will be determined by its characteristic time.

2.5 Calibration procedure

Calibration objects are available from the ruling bodies. These objects have two faces each with different characteristic times, see Fig. 2.

3 Technical details of the resolutions of questions raised concerning effects on the test

3.1 The effects of club loft, mass and shafting

We have run a variety of experiments to assess the likely variations associated with these aspects of club head characteristics. The club's loft is of little consequence when it comes to performing the pendulum test, it is however essential that one ensures that the impact location is as it is intended to be. It has been found that the loft of the club does not affect the characteristic time as measured by the pendulum, although we appreciate that this may not necessarily be so for the cannon test. In essence the pendulum is a true measure of the club head's flexibility.

Figure 2: An image of a calibration club for use with the pendulum device.

The way in which the club head is shafted also does not play a rôle. The most important aspect of the shaft or mounting of the club is its ability to present the club in a consistent fashion for repeated hits. We have also found that some configurations are subject to low frequency oscillations and it is essential that these are stopped before the next impact. This is all so that the correct impact location is hit on the club face. The gripping of the club is also not crucial, in fact it is possible to freely support the club head in front of the pendulum and obtain the same results for the characteristic time. The obvious addendum here is that the head must always be presented in the same way. We add the final caveat that the shafting of the club head should not add excessive amounts of weight to the head, as we are about to discuss that there is an effect associated with club head mass.

In Appendix D we give details of a study into the effect of changing the club head mass. This comprises a theoretical and experimental analysis of the issue. We have demonstrated that increasing the club head mass will increase the characteristic time, although we add that this is a secondary effect. We observe that for an increase of 1g we would expect to observe an increase in characteristic time of 0.25μs. We should set this finding in context and realize that the mass of most club heads submitted to the USGA in 2003 lie between 180g and 215g, with a mean of 197.4g.

3.2 Clubs with coated faces

Within our own analysis and at the suggestion of certain manufacturers we have investigated the effect of coatings on the face. We should start by saying that most of our analysis has concerned coverings which are far thicker than those currently being used, but nevertheless we want to ensure that the technique is robust. We shall start by giving an example of how the pendulum's result for a club head might be affected by the addition of a covering. In essence since the impact velocity of the pendulum is far lower than a real impact the change in characteristic time can be affected by the addition of a coating. As one might expect it has a disproportionate effect on the lower velocity impacts. Applying a coating to the face will cause the characteristic time to increase at each velocity (more so for the lower velocities). This has the effect of artificially increasing the slope B (as currently calculated) and hence ultimately reducing the calculated intercept A. It is quite feasible that the application of a covering will change the characteristic times so that a non-conforming club can be made to appear conforming. This is obviously of concern to us.

In essence the technique we are adopting uses the pendulum's measurements and a further measurement to adjust the model so that the results again correspond to the uncoated tests. This is accomplished by measuring the thickness of the coating and adjusting the value of the exponent k. This adjustment is also based on the original value of the calculated slope B with an exponent $k = 0.33$. We note that we have designed an algorithm whereby this method is only applied where clubs might have changed their conformance status by the presence of the coating. We discuss the details of the procedure and analysis in Appendix E. We note that this technique has been demonstrated as being very successful.

4 Discussion of the test and submission protocol

4.1 Submission process

We shall now discuss the process of club submission. Clubs will be tested for their characteristic time by either the R&A or the USGA depending on where the headquarters of the company resides. The only exception to this would be a club produced solely for use within the other governing body's territory.

Each governing body will have a pendulum, residing in St Andrews, Scotland and Far Hills, New Jersey, USA, respectively. These will be the pendulums which will determine the definitive measurement of conformance for clubs and once a club has been measured on one of these pendulums the acquired decision will be accepted by both organisations. Hence the club will be deemed to exceed or not exceed the limit worldwide.

4.2 Description of the test protocol

A formal test protocol is being circulated, however it seems pertinent to describe the basic method which will be employed. This is based on the algorithm described on page 6.

- The club face's centre will be marked and the club will be mounted within the pendulum.
- The club will tested using nine hits (three at each velocity setting).
- If the calculated slope is less than 20:

  - If the upper limit of the confidence interval is below the limit plus the tolerance (257μs) then the club will be deemed to conform to the pendulum test.

- else the club needs to be tested with a further nine hits and the recalculated value of the intercept will be used to judge conformance (if $A > 257\mu$s then the club does not conform, if $A \leqslant 257\mu$s then the club will be deemed to conform under this test).

- If the calculated slope exceeds 20 the club should be tested with a further 9 hits

  - If the measured values of the characteristic times are all less than 257μs then the club is deemed to conform to this test.[1]

  - If the value of the intercept is greater than 257μs then the club is deemed not to conform.[2]

  Otherwise measure the coating thickness and calculate the value of the exponent $k_{coating}$ and recalculate the value of the intercept, refer to Appendix E. If this value is strictly greater than 257μs then the club will be deemed not to conform.

We should stress that this is purely to determine the results of the pendulum test and the club will still need to be evaluated in other ways before its conformance can be confirmed.

4.3 Details of the equipment

In order to use a pendulum a company needs to enter into a license agreement with either the R&A or the USGA. Copies of these can be obtained by contacting the appropriate organisation . This includes access to the software, drawings of the mechanical parts and an accompanying parts list (including a description of the electronics). It is our intention to provide calibration objects, described in Section 2.5, which will have been measured on one of the two definitive pendulums. We are confident that the variations between machines and operators are well within the allowed test tolerance of 18μs. The calibration object is purely to ensure that the machine is correctly configured.

In addition to the equipment described above in order to completely test the clubs in the methods described herein it may be necessary to acquire a device for measuring the thickness of coatings, see page 35. In order to mark the centre of the face we have been using a acetate sheet marked with

[1] In this case any recalculation of the intercept will still be below the limit, since the intercept is always lower than the measured values.

[2] It is worth noting that any recalculation of the intercept is only likely to raise it further.

a orthogonal set of axes and a circular hole at the origin. The axes are marked at regular intervals so that the centre can be determined. Copies of these templates can be obtained from either governing body (electronically or physically). In order to test club heads (without shafts) we are using a simple reusable shaft which is designed not to add undue effective mass to the club head.

4.4 Comments on the software and user interface

Within the comments from manufacturers, mention was made of the software and the general user interface. At the moment the software has been written to be used by someone who is familiar with the test and the way in which it is run. The new version of the software is, unsurprisingly, targeted at the version of the test described herein. There are a variety of visual checks which can be applied to individual signals (see page 7). To date these are not included within the software. After the club has been hit nine times the data is analysed and any outlying points are identified and the message box suggests which points should be retested (this is done by clicking the corresponding radio button and re-hitting the club - in order to rerun the calculations it is necessary to press the recalculate button).

At the moment a simple version of datalogging is available which saves: the calculated conformance values in `testlog.txt`; the values of the measured times and velocities in `biglog.txt`; and each club's individual data in a filename which depends on the identification used for the club. There are additional files which contain diagnostic information `T`*clubname*`.txt` and `F`*clubname*`.txt` (Fourier spectra). The source code in Visual Basic is available on the signing of a license agreement, together with further details in the user manual.

5 Conclusions

In this report we have given details of the answers to the questions raised by manufacturers and we feel that the consideration of these points has allowed us to develop a fuller understanding of the pendulum and the science behind it. In order to answer the questions we have also undertaken studies of the likely variation associated with the operation of the pendulum and we are confident that this is well within the stated test tolerance of 18μs. We are confident that the algorithm and use of the filtering scheme is robust and reliable.

The change to a more appropriate exponent in the model has resulted in a modification of the limit to 239μs (with the additional test tolerance giving 257μs as the value of the limit plus the test tolerance). We now have a fuller understanding of the effect of club head mass on the test and we are confident that we are able to test clubs with coatings on the face.

A Details of the mathematical methods behind the data analysis

A.1 Fourier series truncation

The mechanical device produces a signal $(t_i, \mathcal{S}_i)$ which spans 1000μs. The indices of the points run from $i = 1$ to $i = N$. The symbol $\mathcal{S}$ is used for the voltage returning from the scope, which is representative of the acceleration. The recommended scope is a 12 bit device sampling at 50MHz (ADC212/50 www.picotech.com, S167 with the inclusive signal conditioner). At this point in this section we shall consider that the data has been extracted correctly. For the sampling rate of the scope we are using this gives $N = 6240$, with a corresponding time step of 0.16μs. An example of a signal from the scope is shown in figure 3. The accelerometer we are using has an intrinsic noise

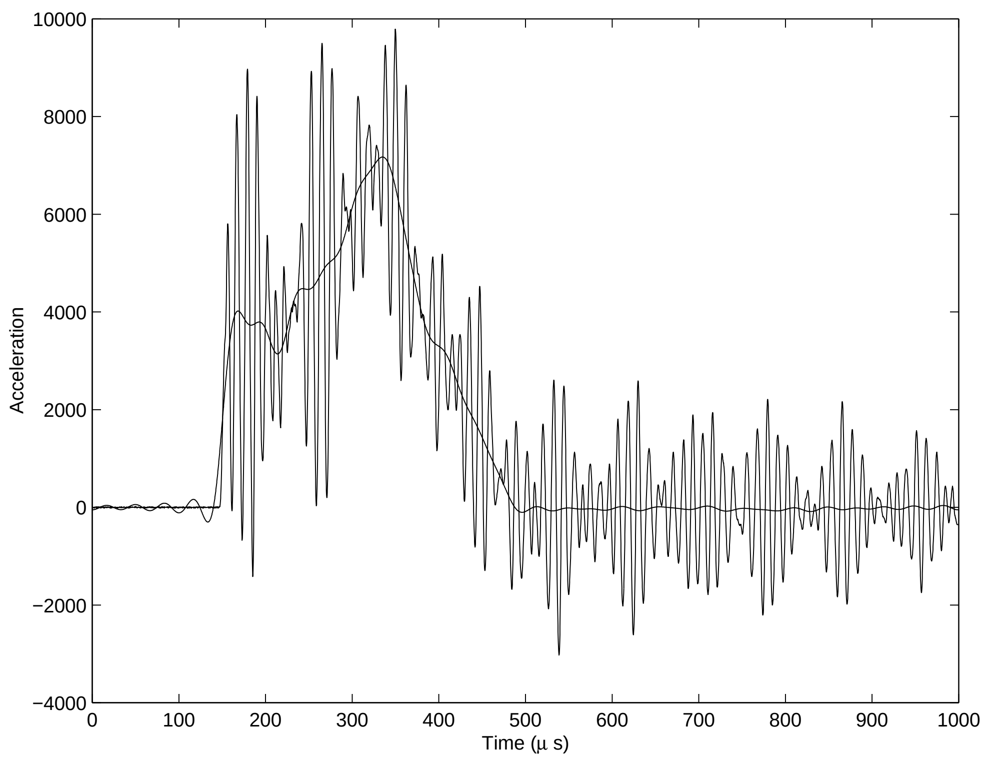


Figure 3: A typical example of the voltage output from the scope. This signal has 6240 points and the resolution is determined by the fact that the scope returns a 12 bit signal. We also include the corresponding filtered signal.

associated with oscillations of the crystal at 65-75KHz (refer to Fig. 4). In order to remove this and any other noise the Fourier spectrum of the data is calculated. Using the period of 1000μs we have $L = 500\mu$s and the Fourier coefficients are

$$a_0 = \frac{1}{2L}\int_{-L}^{L} \mathcal{S}(t)\,\mathrm{d}t$$

$$a_n = \frac{1}{L}\int_{-L}^{L} \mathcal{S}(t)\cos\frac{n\pi(t-L)}{L}\,\mathrm{d}t,$$

$$b_n = \frac{1}{L}\int_{-L}^{L} \mathcal{S}(t)\sin\frac{n\pi(t-L)}{L}\,\mathrm{d}t.$$

Hence the reconstructed signal will take the form

$$\mathcal{S}_{N_m}(t) = a_0 + \sum_{n=1}^{N_m} a_n \cos\frac{n\pi(t-L)}{L} + b_n \sin\frac{n\pi(t-L)}{L}$$

where in the limit $N_m \to \infty$ we will recover the original signal. However, using a finite value of N_m we are able to remove the noise. It has been found suitable to use a value of 50. Extensive testing has been performed to verify that only small changes in the eventual answer are observed with variation in this number. In order to determine the Fourier coefficients it has been found to be sufficient to use a simple Trapezium method of numerical integration. The Fourier spectrum associated with the typical signal shown in figure 3 is shown in figure 4. We note that the acceleration is close to zero at both ends of the signal and consequently we do not encounter Gibbs' phenomenon.

We wish to construct the velocity and this is done by integrating the signal $\mathcal{S}(t)$ from 0 to t, such that the initial velocity is zero. Hence we have

$$v_{N_m}(t) = a_0 t + \sum_{n=1}^{N_m} \frac{a_n L}{n\pi}\sin\frac{n\pi(t-L)}{L} - \frac{b_n L}{n\pi}\left(\cos\frac{n\pi(t-L)}{L} - (-1)^n\right), \quad (1)$$

where the term $(-1)^n$ comes from $\cos n\pi$. We are now in a position to manipulate this signal. Unsurprisingly, the resulting velocity is independent of whether the filtered signal is reconstructed and then integrated, or if it is constructed directly from equation (1), as noted later. In figure 5 we show a representative velocity plot (which again corresponds to the acceleration signal shown in figure 3).

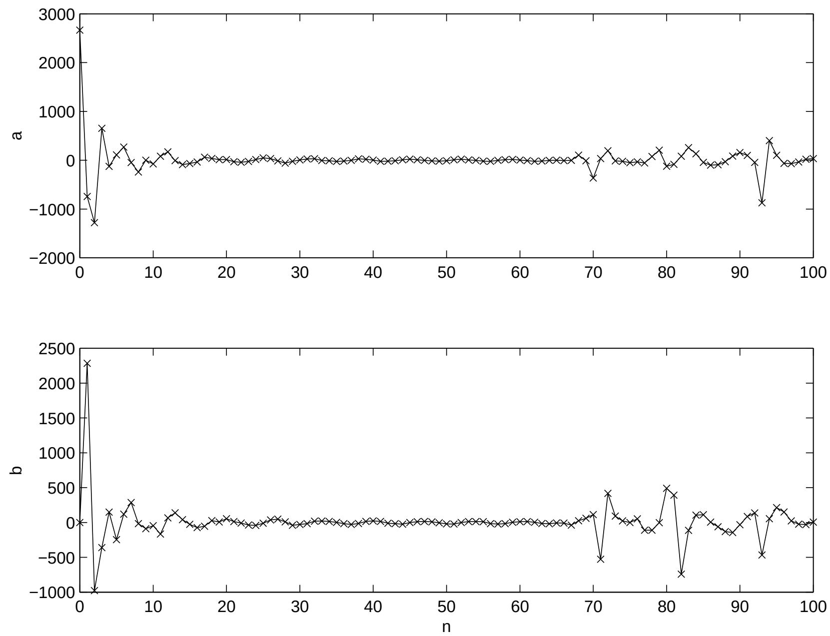


Figure 4: Fourier spectrum for a typical signal shown for the first 100 modes. Here we see evidence of the noise (for $n > 70$ corresponding to 70KHz).

In order to calculate a characteristic time for the club we determine the maximum velocity (simply by looking for the largest value of $v_N(t)$). We shall refer to this quantity as V (which gives a velocity representative of the impact). We have discovered that the most important factor in determining the flexibility of a club is the duration of the impact and our choice of protocol is to eliminate factors which may unduly influence this calculation. We then calculate the time for the velocity to rise from 5% of V to 95% of V, since the identification of the start and end of the signal can be overly sensitive to small changes. In a similar manner the presence of localised vibrations in the club head can make the reliable calculation of the characteristic time troublesome; largely due to the presence of energy in higher modes of vibration. As such it was found to be prudent to fit a cosine function to the rising portion of the velocity profile and this serves to eliminate this phenomenon. For the majority of clubs this makes no difference at all, it is only where the signal contains energy in higher modes, for instance within clubs in which there is some degree of local irregularity near the centre of the face.

The method involves considering the function

$$g(t) = 1 - 2\frac{v_N(t)}{V}$$

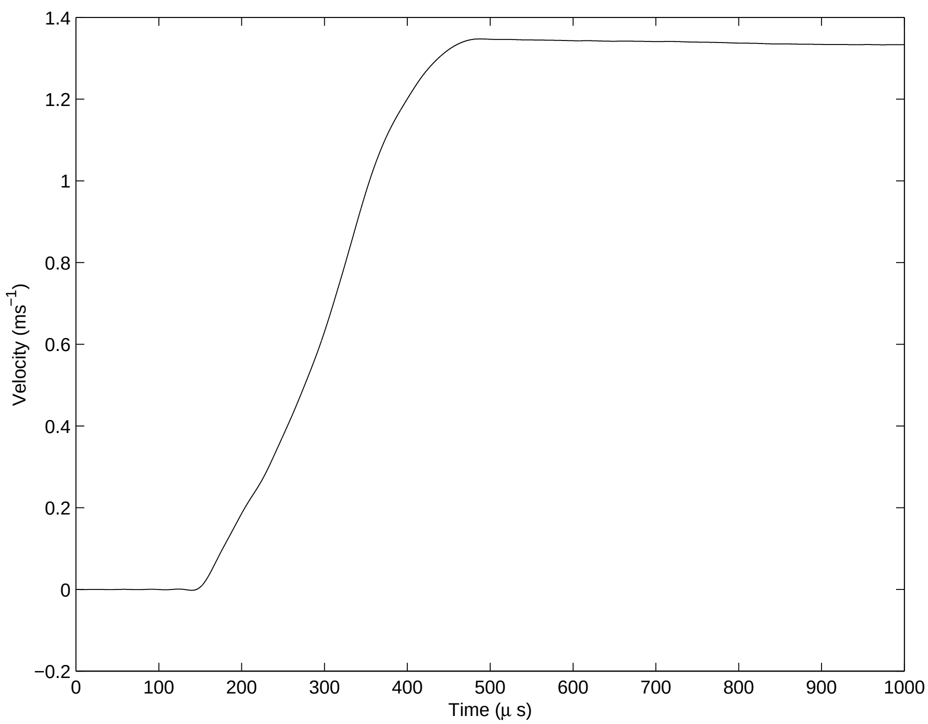


Figure 5: Sample velocity profile for an impact between a club and a ball. Notice that since we are using a longer signal collection time and a corresponding longer pre-trigger this gives us a longer plateau region at the start and in fact also at the end.

(which runs from 1 to -1 over the range of interest). We now seek a function $\cos(\alpha t+\beta)$ and rely on linear regression to minimise the quantity

$$\mathcal{E}=\sum_{i=j_1}^{i=j_2}\left(\alpha t_i+\beta-\cos^{-1} g(t_i)\right)^2$$

where j_1 and j_2 represent the start and end of the rising portion of the curve[3]. These values are further refined by minimising the function

$$\mathcal{E}_{\cos}=\sum_{i=j_1}^{j_2}\left(\cos(\alpha t_i+\beta)-g(t_i)\right)^2$$

by using a quasi-Newton method. Since we now have an analytic form for the signal in the region of interest we can calculate the times corresponding to $v_N(t)=0.05V$ and $v_N(t)=0.95V$ directly. When $v_N(t)=\delta V$ then $g(t)=1-2\delta$ and hence $t_\delta=(\cos^{-1}(1-2\delta)-\beta)/\alpha$. We now observe that

$$t_{\text{char}}=t_{0.95}-t_{0.05}=\frac{1}{\alpha}\left(\cos^{-1}(1-2(0.95))-\cos^{-1}(1-2(0.05))\right).$$

We conclude this section by noting that the method has been designed so that it is insensitive to changes in the algorithm, in the current method we adopt the protocol:

- Acquire signal, filter using a truncated Fourier series, directly integrate to obtain the velocity from the Fourier series

However, one could also

- Acquire signal, integrate using a standard technique (Trapezium, Simpson's 3/8th) and then filter using the truncated Fourier series (in this case one would need to use an even reflection of the signal to remove Gibbs' phenomenon, essentially using a cosine series)

- Acquire signal, filter using a truncated Fourier series, invert the transform and then integrate using a standard method.

The results given by any of these methods differ by less than a microsecond. We should say that the Fourier filter was adopted since we are aware of

[3]The method has been demonstrated to be insensitive to these choices, due to the minimisation of the quantity $\mathcal{E}_{\cos}$. We note that the minimisation of $\mathcal{E}$ and $\mathcal{E}_{\cos}$ are similar but subtly different. The advantage of the operation of minimising $\mathcal{E}$ is that this is a direct calculation whereas $\mathcal{E}_{\cos}$ has to be minimised using an iterative procedure.

the frequency of the noise we wish to eradicate, although alternatives were considered. It is essential that the filtering does not introduce a phase shift into the data. The adoption of the 5-95% methodology was to remove the difficulty in determining when the system had reached a maximum, which would be far more sensitive to any remaining noise in the system.

A.2 Statistical analysis of the data

The data points are analysed using the method of deleted Studentised residuals for both the nine and eighteen point tests. The leverages are also calculated to ensure that no significantly different velocities have occurred. We note that this is only likely to arise where the test has not been performed correctly.

The calculation of the confidence intervals is performed using standard analysis which can be found in most standard texts on data analysis (for instance 'The Statistical Analysis of Experimental Data' John Mandel, Dover, ISBN: 0486646661).

B Testing Hertz Contact Assumption of the Pendulum Test

B.1 Introduction

One of the main central features of the pendulum test is the hypothesis that the total characteristic time may be separated into two contributions, that due to the assumed linear stiffness of the club head and that due to the stiffness of the Hertzian contact between the steel pendulum mass and the face material of the club head.

To date, pendulum results have been provided for actual club heads where, in general, the characteristic time is dominated by the linear stiffness of the club head. In order to test the Hertzian contact model, blocks of solid metal were tested.

B.2 Method

Two blocks of metal, one 6061-T6 aluminium and the other a steel of unknown alloy were shafted using a removable shaft. The impact spot was directed near the centre of gravity of the block. Before conducting the pendulum test, the pendulum mass was released three times into the block. It was noticed that a small permanent indentation was left in the blocks.

The pendulum test was then conducted on the two blocks. Three impacts were recorded at each of the three release heights starting with the highest release point, followed by the middle release point and ending with the lowest release point. This test was then conducted a second time for a total of eighteen impacts.

Following the pendulum testing, the permanent indentations in the blocks were measured using a contour tracing machine magnifying the indent by a factor of two hundred.

B.3 Results

It was found that the aluminium block had a permanent indentation having a radius of 62 mm and the steel block had a radius of 77 mm. This permanent radius was then used, along with the material properties of the blocks and the pendulum mass in the numerical simulation of the pendulum test.

The results of both the experiment (data points) and the simulation (solid line) are shown in Figure 6. It may be seen that there is good agreement between the experiment and the simulation.

It should be noted that since the blocks are clearly rigid except for the Hertzian contact, the non-linear exponent (k) is equal to 0.20 as opposed to the value of 0.33 used for flexible club heads.

When a linear fit is applied to the experimental and numerical simulation data given in Figure 6, it can be seen that

(i) there again is very good agreement and

(ii) the intercept is nearly zero.

It may be concluded therefore that the assumption that the pendulum test isolates the club stiffness from the Hertzian contact is valid.

	Aluminium Intercept	Slope	Steel Intercept	Slope
Numerical	-0.3	62.5	1.7	46.3
Experiment	-5.4	64.9	-3.2	52

C Velocity dependence of the Pendulum Test

C.1 Introduction

The pendulum test has been successfully modelled with a one-dimensional non-linear club model comprised of a linear spring representing the club

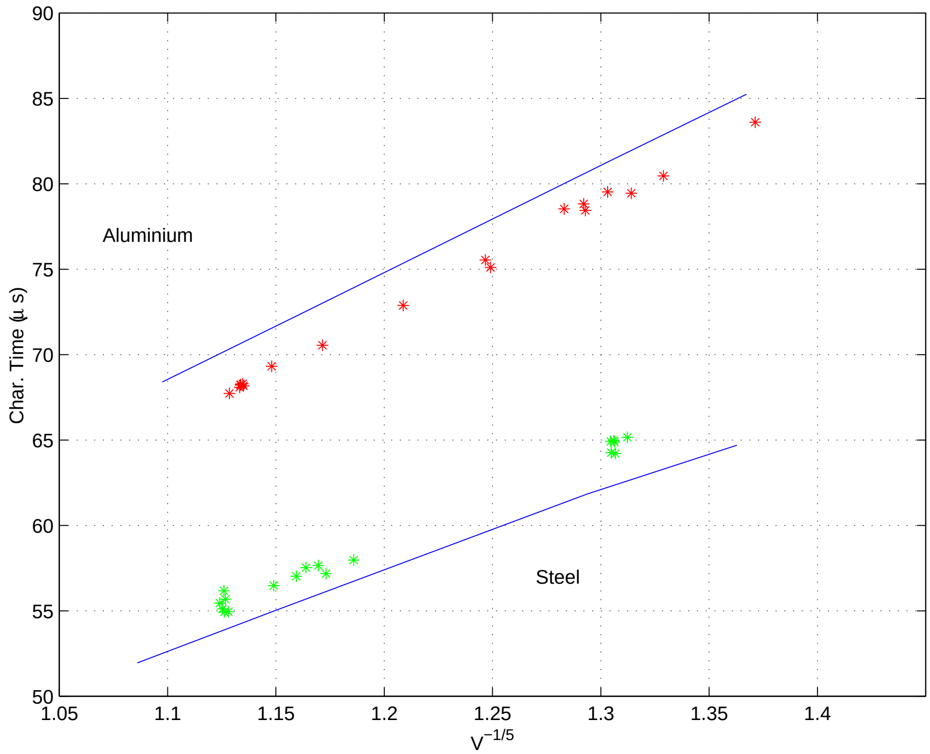


Figure 6: Results of experiment and numerical simulation for blocks of aluminium and steel.

head flexibility and a non-linear spring representing the Hertzian contact between the club face and the pendulum mass. The model provides a basis for isolating the local deformation flexibility from the overall flexibility of the club head (which is the feature of interest) by testing at multiple impact velocities.

As described in previous documents, the current procedure calls for linearizing the characteristic times as a function of the impact velocity using the classical Hertzian relationship between contact time and impact velocity:

$$T = A + BV^{-1/5}.$$

However, the value of the exponent (-0.2) is strictly true only for the case where the club head is infinitely stiff. For realistic club flexibilities, the most appropriate value for this exponent depends on both the club face material and the underlying flexibility of the club head. It has been found that a value of -0.33 adequately covers the range of face materials for clubs with flexibilities near the conformance limit.

C.2 Test Model

It has been found that a simple model of the club head is sufficient to predict the predominant behaviour of the pendulum test. The pendulum mass is represented simply as a point mass having no elastic properties. Deformation of the pendulum mass is contained in an interface spring associated with the club.

The club head is modelled as a single mass and two springs. The first spring (K_c) is linear and is intended to represent the overall flexibility of the club head. This flexibility, along with the mass properties of the club head contribute to the intercept value that is isolated by the pendulum test.

In addition to the overall flexibility of the club head, a second spring is included to model the localized contact between the pendulum mass and the club face. Hertzian contact definition and hence this spring is non-linear and their derivations are well described by several sources. For example, Johnson (1985)[4] provides the following definition for the non-linear spring:

$$k_H = \frac{4}{3}(R^*)^{\frac{1}{2}} E^*(x_H - x_m)^{\frac{1}{2}}$$

where $(x_H - x_m)$ includes the compression of both the club face and pendulum mass at the localized area of contact. The model parameters are defined as:

$$R^* = \frac{1}{\frac{1}{R_b} + \frac{1}{R_c}} \text{ and } E^* = \frac{1}{\frac{1-\nu_b^2}{E_b} + \frac{1-\nu_c^2}{E_c}}$$

[4] Johnson, K. L. (1985) *Contact Mechanics*, CUP, UK

where R_b and R_c are the radii of curvature of the pendulum mass and club face respectively and ν and E are the Poisson's ratio and modulus of elasticity.

Owing to the non-linear nature of the model, the equations of motion must be solved numerically. In general terms, the equations of motion are solved and the resulting velocity change of the pendulum mass is recorded.

C.3 Results

It may be shown that the characteristic time for a club head depends on both the linear and non-linear springs of the model, and the initial velocity of the pendulum mass. However, owing to the non-linear nature of the Hertz contact, as the impact velocity increases the characteristic times of various face materials converge (for clubs with the same linear flexibility). **It is through this feature that the pendulum test isolates the linear flexibility of the club head and conformance is determined.**

Using the model described above, various club stiffnesses and face materials were analyzed over a wide velocity range. Club stiffness values ranging from 20 MN/m (highly conforming) to 3 MN/m (highly nonconforming) and face elastic modulus values ranging from 200 MPa (steel) to 3 MPa (polymer) were considered. In all cases, the pendulum mass material was steel with a radius of curvature of one inch. The radius of curvature of the club head is assumed to be ten inches.

For each combination of club stiffness and face material, the characteristic time is plotted against the velocity change of the pendulum mass. A typical plot is shown in Figure 7.

It has been found that the function:

$$T = A + BV^{-k} \tag{2}$$

fits the data very well over a wide range of velocities and the asymptotic nature of the function agrees with the physics of the equations of motion. It has been found that the intercept (A) is a function primarily of the linear club stiffness (with a small influence by the mass of the club head within the range of driving clubs). Therefore, by fitting experimental data to function (2), a relative measure of the club flexibility may be made. The intercept (A) is referred to as the infinite velocity characteristic time since the value of the function at the intercept is the characteristic time for an infinitely high impact velocity. The slope (B) indicates the magnitude of the velocity dependence of characteristic time and is primarily a function of the club face material and club stiffness. High values of the slope indicates soft face materials.

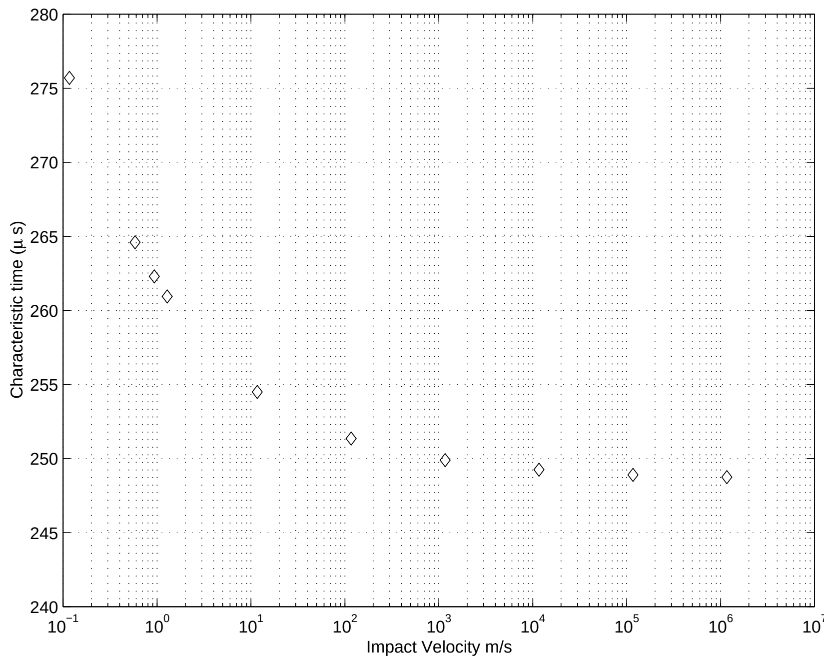


Figure 7: Characteristic Time velocity dependence.

Figure 8 shows a plot of the data given in Figure 7 but with the velocity raised to the power -0.329. The value of the exponent for this set of data was generated by minimising the error between the model generated data and function (2).

For the data plotted in Figures 7 and 8, the intercept value of 248.6 μs is indicative of a marginally conforming club and the slope of 13.7 is indicative of a metallic club face.

A similar procedure of finding the optimum value of the non-linear exponent for the range of club stiffness and face materials was used. The results are given in Table 1.

Three options are now available for the practical application of data analysis:

i. Fit a function over the range of data in Table 1. A preliminary function that fits the data well is:

$$k = \exp\left(-(1.07 + 87.6A^{-1.5} + 0.00099B)\right). \tag{3}$$

For a set of experimental data, one would first assume a value for k, find the slope and intercept then use (3) to calculate a new k and iterate until the solution converges.

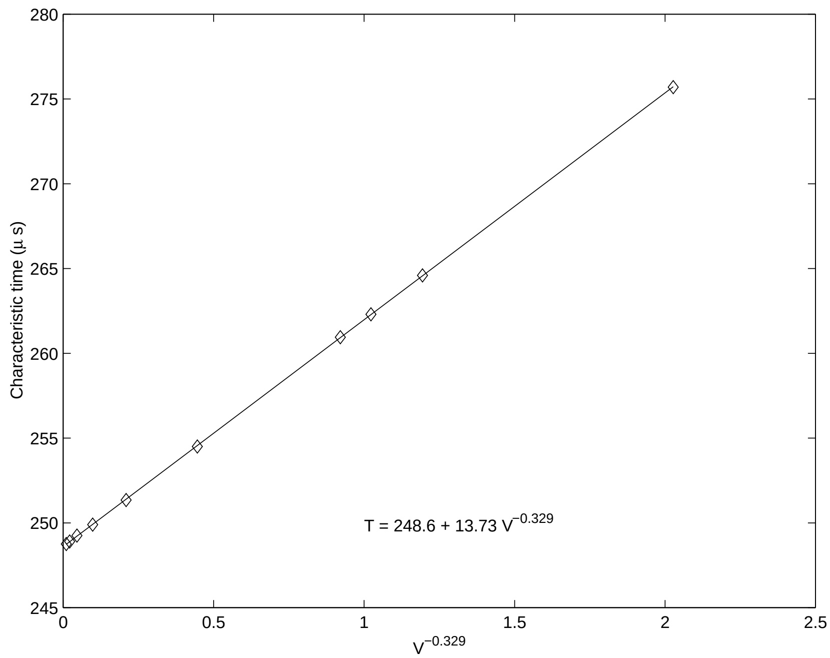


Figure 8: Linearised Characteristic Time velocity dependence.

ii. Since we are generally interested in conformance, concentrate on the variability of k near the conformance limit intercept and make k a function of slope only. The dependence of k on slope at the conformance limit is shown in Figure 9. In this case, the procedure would be similar to that in (i.) but with the exponent being adjusted based on slope alone. i.e.,

$$k = 0.3331 - 0.0003B.$$

iii. Select a constant value of the exponent that is most applicable and that does not have an undue influence on any club. Table 1 shows that a value of -0.33 is applicable for metallic faced clubs and is suitable for all others. The largest expected error using a value of -0.33 is 4 microseconds for a club with a very soft face having a slope of over 80, refer to Figure 10.

C.4 Recommendation

A constant exponent value (-0.33) minimises the expected error on the intercept value for metallic clubs, does not have an undue influence on low modulus faced clubs and does not complicate the data analysis procedure. It

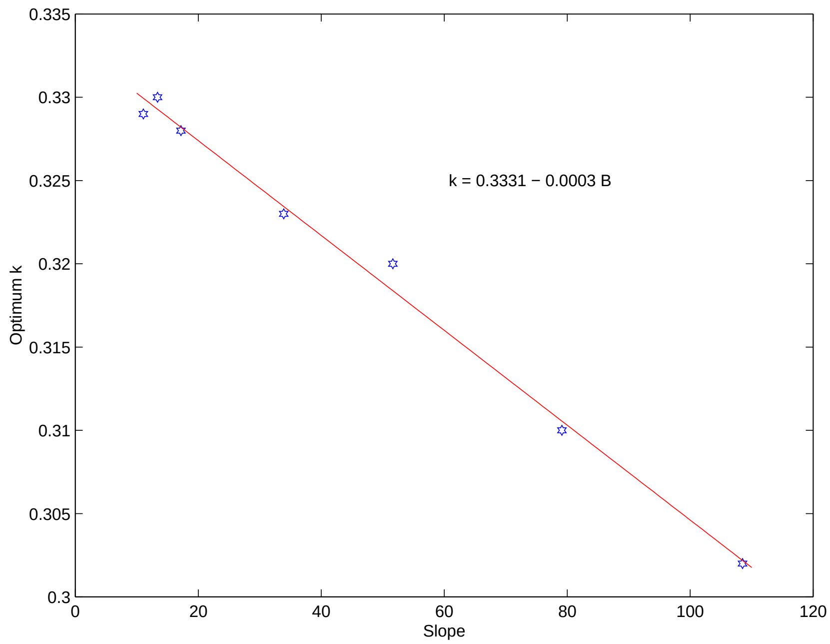


Figure 9: Dependence of optimum exponent on slope at conformance limit.

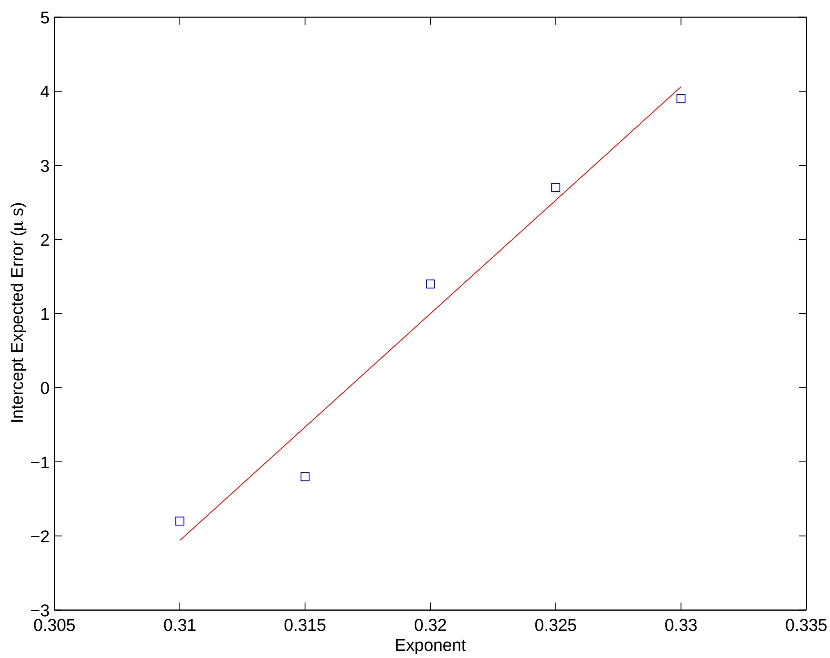


Figure 10: Effect of constant non-linear exponent on intercept error for a club with a slope of 80.

γ		K_c 20	15	12	10	8	7.1	5	4	3
	A	148.3	171.2	191.2	209.5	234.2	248.6	296.24	331.2	382.5
200	k	0.325	0.326	0.328	0.329	0.328	0.329	0.331	0.331	0.334
	B	15.50	14.10	13.10	12.35	11.53	11.06	9.85	9.14	8.28
120	k	0.324	0.325	0.327	0.328	0.329	0.329	0.330	0.331	0.331
	B	18.62	17.05	15.84	14.44	13.89	13.38	11.91	11.07	10.08
69	k	0.320	0.323	0.325	0.326	0.327	0.328	0.329	0.329	0.330
	B	23.93	21.86	20.34	19.17	17.89	17.18	15.33	14.26	12.96
20	k	0.310	0.314	0.317	0.319	0.321	0.323	0.325	0.327	0.327
	B	46.55	42.77	39.95	37.78	35.24	33.90	30.36	28.26	25.80
10	k	0.301	0.306	0.310	0.312	0.315	0.317	0.321	0.323	0.325
	B	69.80	64.42	60.36	57.31	53.61	51.65	46.31	43.18	39.40
5	k	0.289	0.295	0.300	0.303	0.308	0.310	0.315	0.318	0.321
	B	104.9	97.57	91.84	87.44	81.89	79.14	71.32	66.62	60.94
3	k	0.279	0.286	0.291	0.295	0.300	0.302	0.309	0.312	0.316
	B	141.1	131.9	124.8	119.1	112.1	108.5	98.14	92.00	84.37

Table 1: Variation of optimum exponent. The variables are: γ the elastic modulus (GPa), A the intercept, B the slope, K_c the club linear stiffness (MN/m) and k the exponent.

is therefore recommended that the current proposed procedure by revised to use an exponent of -0.33 rather than -0.2 to linearize the experimental data.

D Effect of club head mass on Characteristic Time

The flexibility of the club head is the predominant parameter affecting the characteristic time for a club head. The club mass is a secondary effect. The following quantifies this effect for clubs near the conformance limit using a theoretical model described in previous documents. Experimental data is also provided which shows good agreement with the theoretical data.

Figure 11 shows the distribution of club head mass for those driving clubs submitted to the USGA in 2003. It may be seen that the vast majority of clubs fall between 180 and 215 grams (99.5%). The average club head mass is 197.4 g.

Figure 12 shows the infinite velocity characteristic time for a theoretical club head near the conformance limit ($K_c = 8.0$ MN/m) over the expected range of club head mass (180 to 215 g).

It may be seen in Figure 12 that a change of 1 gram of club head mass will result in an expected change of characteristic time by 0.25 μs (for clubs near the conformance limit).

As a check on the model results, two club heads near the conformance limit have been measured for characteristic time with mass added incrementally. Figure 13 shows the experimental values compared to the theoretical expectation of the effect of mass. In each case, the model club flexibility was adjusted in order that the infinite velocity characteristic times match before mass was added using lead tape. It may be seen in Figure 13 that there is good agreement between the experimental and the predicted mass effects.

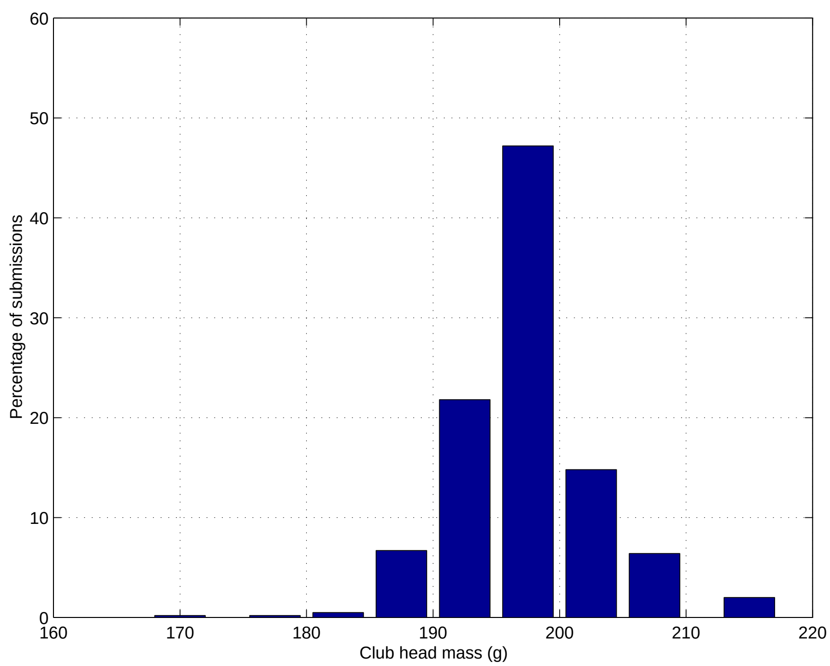


Figure 11: Distribution of club head mass.

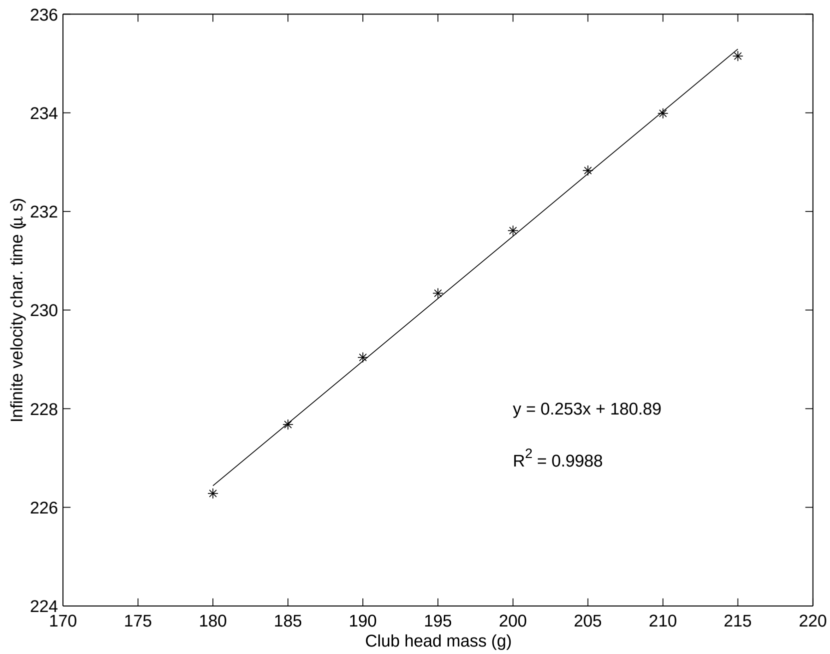


Figure 12: Theoretical effect of mass on characteristic time.

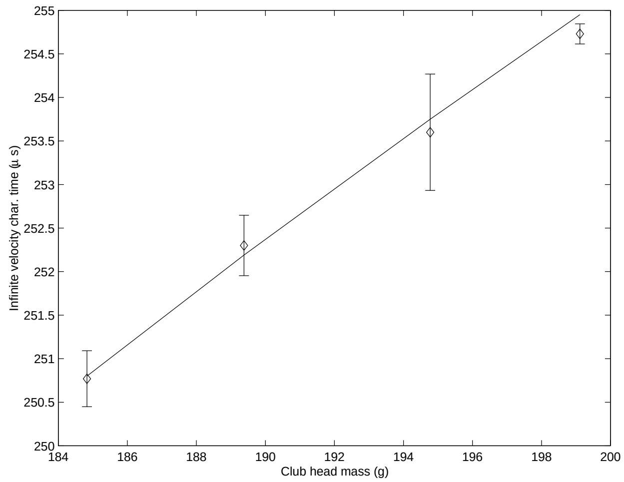
D.1 Conclusions

Over the expected range of driver club head mass, there is a direct but minor effect of mass on the characteristic time. It is not believed that this effect warrants the difficulty involved with obtaining the club head mass which would almost certainly involve disassembling the club head.

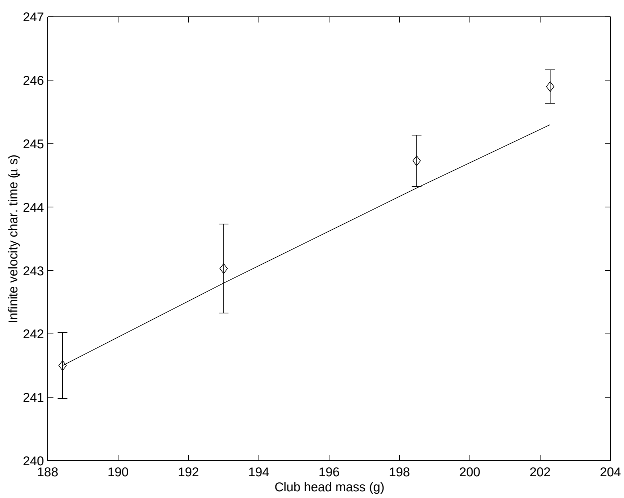
E Testing of coated clubs

We start by showing an example of how the application of a polycarbonate coating could effect the pendulum test. We appreciate that the thickness and type of coating described herein will not be durable within real impacts but nevertheless we feel it acts as a good illustration of the phenomenon. We shall give results for other coatings in due course

Polycarbonate Thickness(μm)	Fit
Uncoated	$y = 248.72 + 7.08V^{-k}$
248.9	$y = 233.18 + 36.9V^{-k}$
497.8	$y = 235.72 + 44.02V^{-k}$
749.3	$y = 225.66 + 60.46V^{-k}$



(a) Club A



(b) Club B

Figure 13: Effect of adding mass artificially to a club head, two examples. Each figure shows the theoretical prediction and the actual measurements from the pendulum.

As can be seen we have managed to reduce the value of the intercept from 248.72μs to 225.66μs merely by adding a coating (this coating has been demonstrated to have no effect on the cannon test).

We make the observation that most paints we have measured have been less than 60μm thick. We also fully appreciate that a coating of the type above is likely to have other effects on the movement of the ball. We have applied a model to the solution of this problem and found that this adequately captures the physics of the problem. The model includes a compliant layer above a substrate and in essence our findings are that: for a thin coating the club behaves as if it is uncoated, as the coating thickness increases the club's velocity dependence is changed by the presence of the coating and finally for a very thick coating the nature of the substrate becomes virtually irrelevant. In these three régimes we find that $k = 0.33$ adequately models the velocity dependence of the impact in the first and last cases, and for the second case the value of k depends on the coatings thickness and its flexibility (reflected in the value of the slope B).

At this point we add the comment that the application of a coating will artificially increase the slope B and as such we can detect whether a coating is present using the value of B. If the slope is below a certain threshold (namely 20) then we deem that the test using a value of the exponent of 0.33 adequately captures the physics of the impact. If the slope exceeds this value then either the club is behaving as a plastic or persimmon club, or it is coated. We seek to differentiate between these two cases using the thickness probe.

The thickness probe automatically detects whether the club is ferrous or non-ferrous (or more specifically whether the club is magnetic or not). In the former case it uses magnetic inductance to measure the thickness of the coating and in the latter case it uses an eddy. For plastic or persimmon clubs the instrument fails in that there is effectively no substrate and in these cases the results of the pendulum test stand. We note that this may also occur for clubs with excessive coating thicknesses on them and again the pendulum result is an accurate reflection of how the club would behave in the cannon test and thus in a real impact. We note that the thickness of the coating may be measured in other ways if necessary.

The scope we are using is an Exacto/FN/e scope, for more information see

`http://www.elektrophysik.de/e/p_exacto.html`

We shall describe in detail the way in which the club is tested in the section on the outline of the testing protocol. Here though we mention that the value of the exponent is given by the formula:

$$k_{coating} = 0.33 + \beta(h)\,(\ln(B) - \ln(13.34))\,,$$

where B is the slope calculated from the original data and $\beta(h)$ will be given in due course. The value of 13.34 corresponds to a slope of a typical club near the limit with a particular set of parameters chosen to characterise the club. The value of the gradient $\beta(h)$ can be found from the table:

Coating thickness (μm) h	Gradient $\beta(h)$
< 63.5	0
127	0.0752
254	0.0965
381	0.1157
508	0.1176
635	0.1091
762	0.0928
1016	0.0643
1270	0.0445
1524	0.0327
2032	0.0183
> 2032	0

We note that the value of $\beta(h)$ at intermediate values of h can be found by suitable interpolation (either linear or cubic). We note the phenomenon that as the thickness increases the value of k will ultimately return to 0.33, corresponding to a coating which is so thick that the behaviour has dominated over that of the substrate. To set this within context five business cards are approximately 1790μm thick. In figure 14 we show the values of $k_{coating}$ acquired in this way.

We now show results obtained using this method

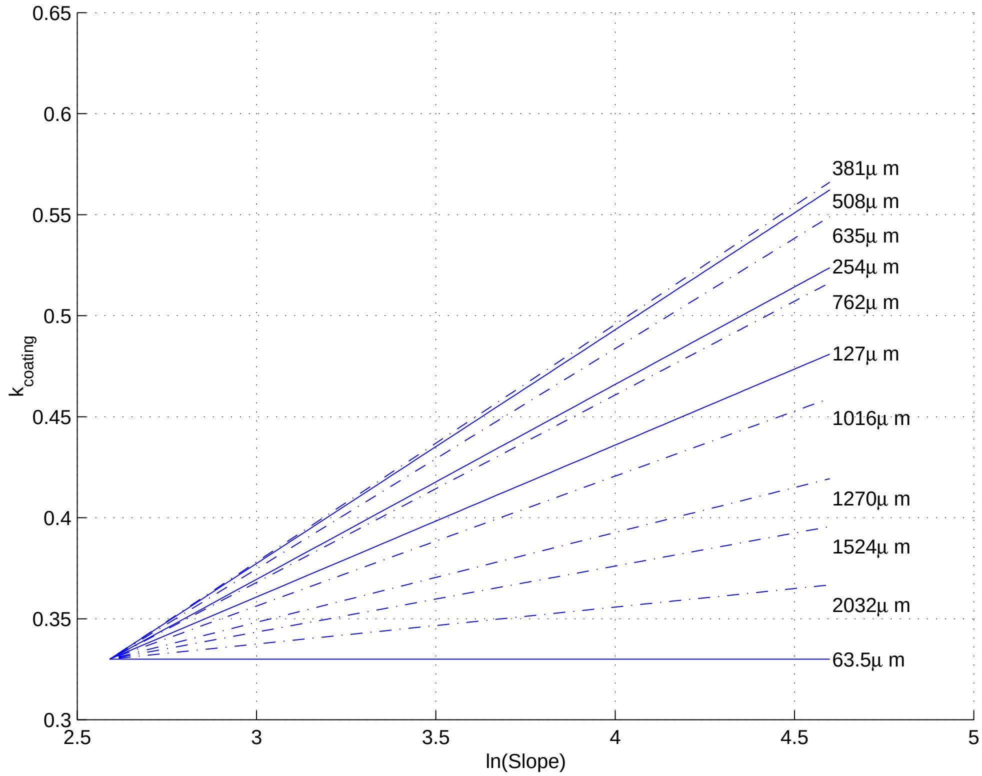


Figure 14: Variation of $k_{coating}$ with coating thickness and slope. Dashed lines are used for thicknesses greater than 500μs. Note that as the coating thickens the value of k returns to value of 0.33.

Coating	Intercept with $k = 0.33$ (μs)	Error(μs)	Intercept with variable k (μs)	Revised Error (μs)
Uncoated	248.8	0.0	248.8	0.0
109.2μm Polycarbonate	246.6	2.2	246.6*	2.2*
248.9μm Polycarbonate	233.7	15.1	245.3	3.5
375.9μm Polycarbonate	233.2	15.6	249.4	-0.6
497.8μm Polycarbonate	236.3	12.5	251.8	-3.0
749.3μm Polycarbonate	226.5	22.3	246.0	2.8
396.2μm Delrin	241.2	7.6	248.7	0.1
500.4μm Delrin	235.0	13.8	246.3	2.5
645.2μm Delrin	234.1	14.7	246.1	2.7
838.3μm Delrin	232.7	16.1	245.1	3.7

In the case of the second row (marked with an asterisk) the slope was below the threshold of 20 so the algorithm was not applied.