Prediction of Initial Velocity from COR and Contact Time

July 20, 2012

1. Summary

It has been demonstrated that the measure of Initial Velocity, as measured by the USGA's Illinois Tool Works (ITW) machine, can be accurately predicted by measuring the normal rebound velocity and contact time resulting from an infinite mass impact test. Predictions of Initial Velocity are accurate to within ±0.4 ft/s at a 95% level of confidence. The model was developed using a population of golf balls having performance uniformly distributed over the full range of both Initial Velocity and contact time. The robustness of this model was validated through a bootstrapping analysis of this data included in this discussion.

2. Test Methods

A sample of golf balls having a wide range of both coefficient of restitution (e) and contact time (t_c) were obtained. The normal coefficient of restitution (e), normal contact time (t_c), and Initial Velocity data were all collected within less than two days of each other to minimise any possible degradation of ball performance. The normal impact measurements were made by firing balls from an air cannon (tilted downward at 5 degrees below horizontal) into a massive vertical block (i.e. an infinite mass). All balls were maintained in a temperature and humidity controlled environment at all times prior to tests.

The normal velocity before and after impact were measured by pairs of lasers ballistic screens. The design impact velocity was 143.8 ft/s, matching the impact speed of the ITW device. The impact force was measured with a 3-axis force transducer (in this case a Kistler type 9067). A schematic of this test apparatus is shown in Figure 1.

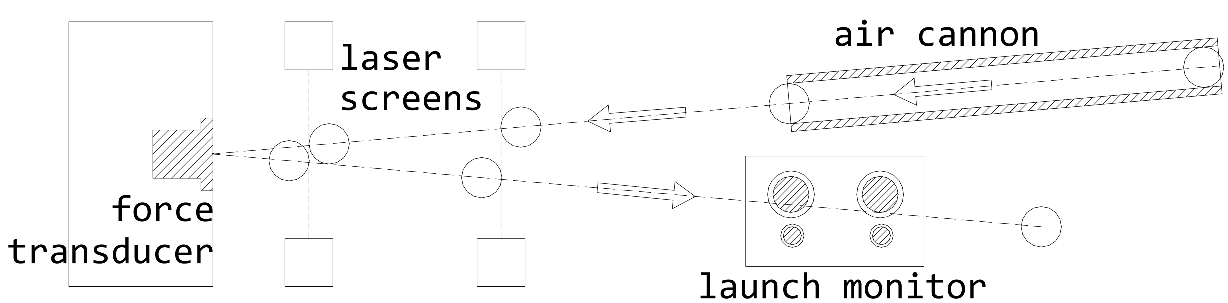

Figure 1: Normal COR and contact time test apparatus schematic drawing.

Measurement of contact time was made by analyzing the signal from the normal force time history output from the force transducer. Unwanted high frequency noise in the force transducer output signal is removed by constructing the full Fourier spectrum of the data and then retaining only the primary lower frequency information (f_{cutoff} = 3 kHz). The end of the contact time (duration) is defined here as when the normal force returns to 5% of the maximum normal force. Figure 2 is a sample of a filtered force time history used to calculate contact time. Figure 3 is a plot of contact time, t_c, versus the sum of normal inbound and outbound COR test speeds, V_e, for the full population of ball models used in this analysis. Each data point in Figure 3 represents the average of a 12 shot test.

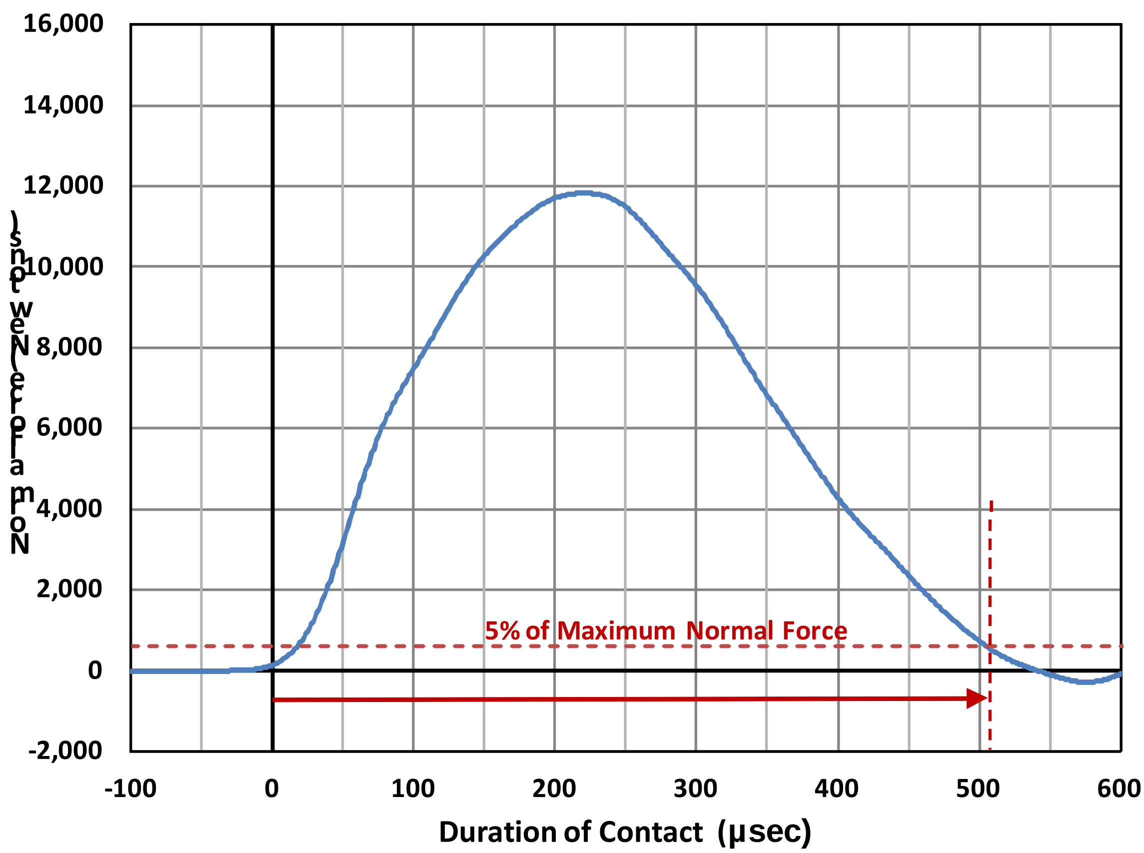

Figure 2: Sample of filtered transducer output of normal force used to calculate contact time (duration).

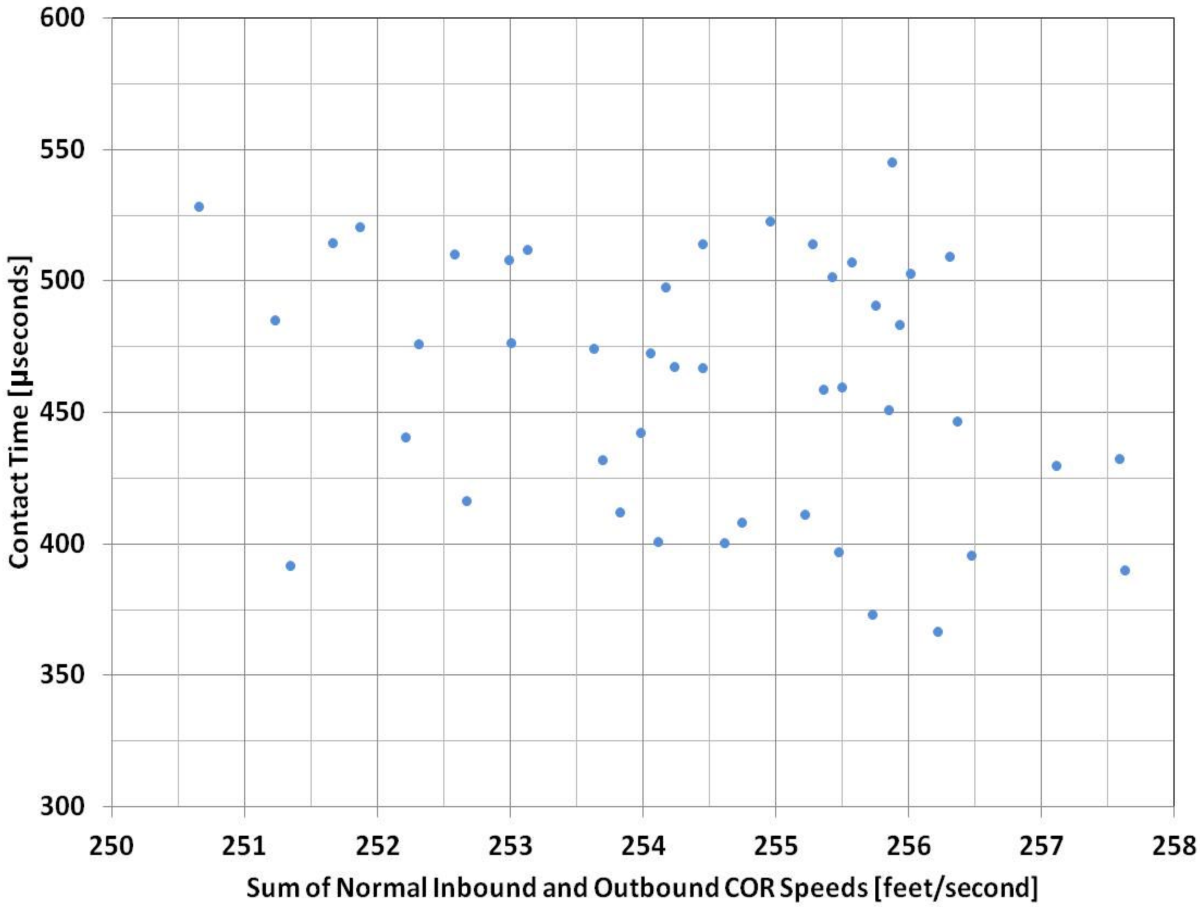

Figure 3: Contact time versus sum of normal inbound and outbound COR speeds.

The initial rationale for this research was to determine how well the results of the normal ball COR test could directly predict the Initial Velocity results on the ITW machine. Figure 4 is a plot of actual ITW Initial Velocity versus Initial Velocity as predicted from the COR test data. The predicted Initial Velocity values from the COR test (V_e) are simply the sum of the normal speeds inbound and outbound. The normal velocity components were calculated from the actual rebound angle measured by the launch monitor depicted in Figure 1. The differences in the data from the simple COR test prediction of IV minus actual Initial Velocity, range from 0.3 to 6.0 ft/s and all are biased to over predict the actual Initial Velocity. The average difference between the Predicted Initial Velocity and the Actual Initial Velocity was 3.1 ft/s and the median difference was 2.7 ft/s.

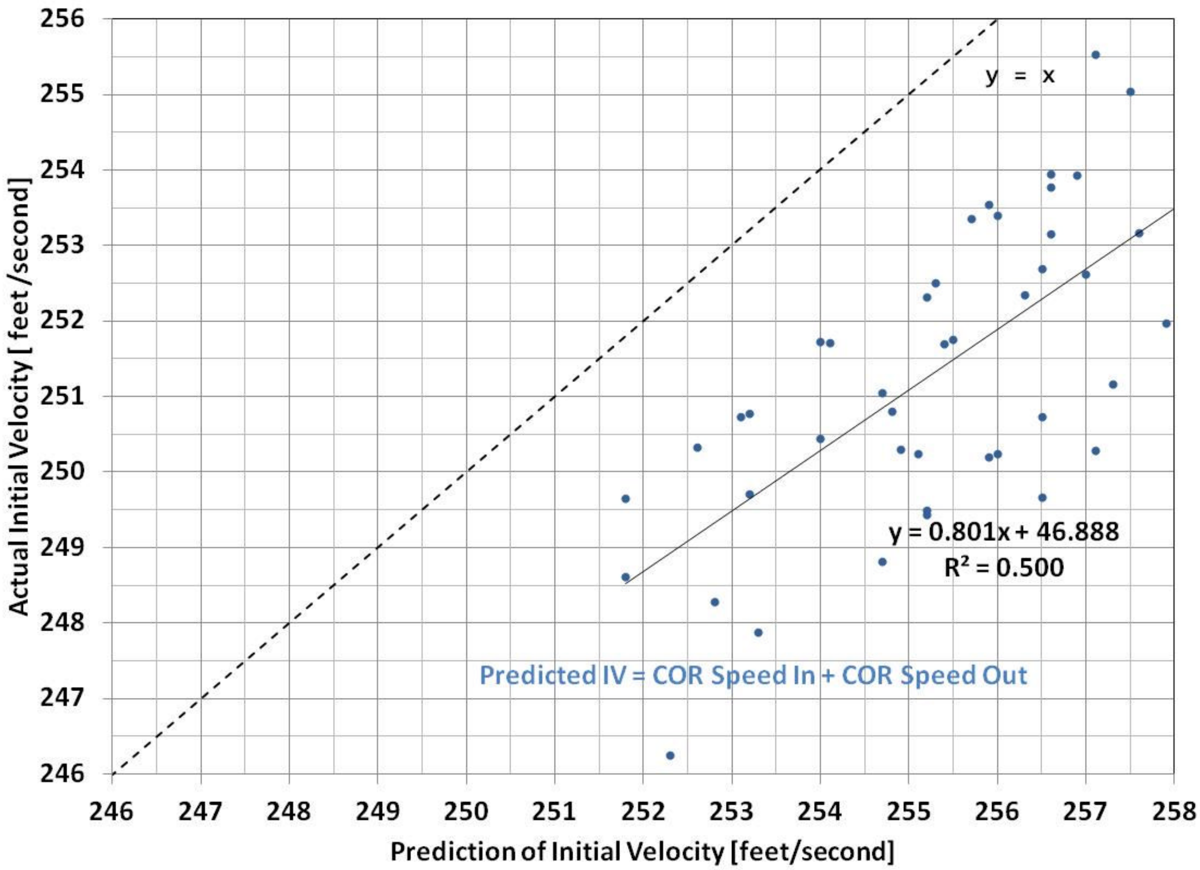

Figure 4: Actual Initial Velocity vs. Prediction of Initial Velocity from Normal Rebound Speed.

To improve the prediction of the Initial Velocity, the normal impact contact time (t_c in μs) and the COR test equivalent of Initial Velocity (V_e) were used to predict actual Initial Velocity (IV). The resulting prediction (regression analysis done with Minitab) of Initial Velocity is the following function:

$$IV\ (V_e,\ t_c) = -19.502 + 1.006\ V_e + 0.030\ t_{c.}$$

The inclusion of contact time (t_c in μsec.) from the normal impact test improved the prediction of Initial Velocity from just the simple model of using the sum of the inbound and outbound COR test speeds. The bias between the predicted Initial Velocity and the Actual Initial Velocity has been incorporated into a constant term and the slope of the regression between actual and predicted is unity. The average absolute difference between the actual and predicted is 0.14 ft/s with the median value being 0.11 ft/s (R^2 = 0.99).

Figure 5 is a plot of the Actual Initial Velocity measured versus the Prediction of Initial Velocity as a function of normal contact time, t_c, and the sum of normal inbound and rebound speeds (V_e). Figure 5 also includes the 95% prediction intervals. The average upper and lower bounds of the 95% prediction interval are approximately ±0.39 ft/s.

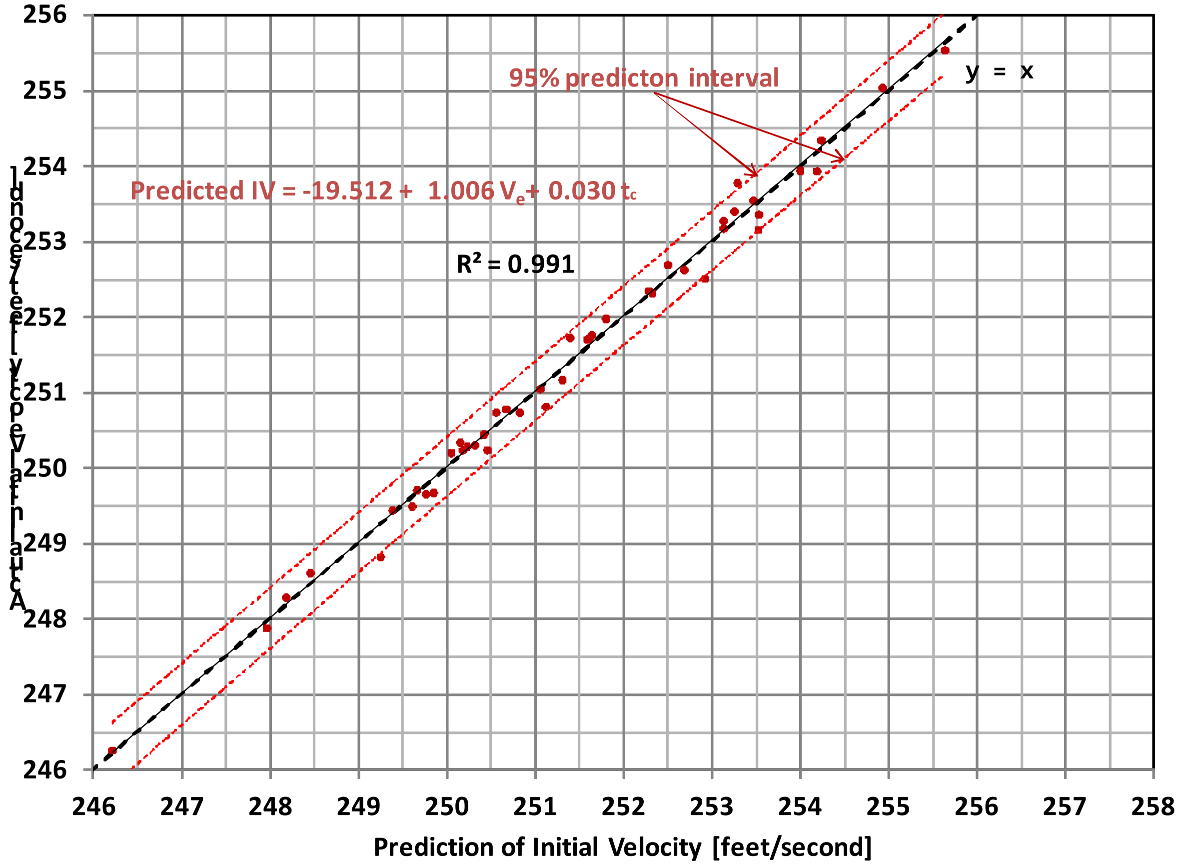

Figure 5: Actual Initial Velocity vs. Prediction of Initial Velocity as a function of contact time and normal rebound speed.

3. Statistical Analysis of Data

To evaluate the robustness of the regression model developed in this discussion, a bootstrapping analysis of the data was conducted. The analysis consisted of randomly selecting 20 sets of test results from the full 45, developing a regression model, and testing the non-included 25 test results from this new regression model. This random selection of 20 data sets was repeated 200 times and the residuals from the models for each of the 200 iterations were assessed.

A histogram of the standard deviation of the residuals of the 200 iterations is included in Figure 6. The mean standard deviation of the residuals of the 200 permutations was 0.19 ft/s with the maximum standard deviation of the residuals of the 200 permutations being 0.24 ft/s.

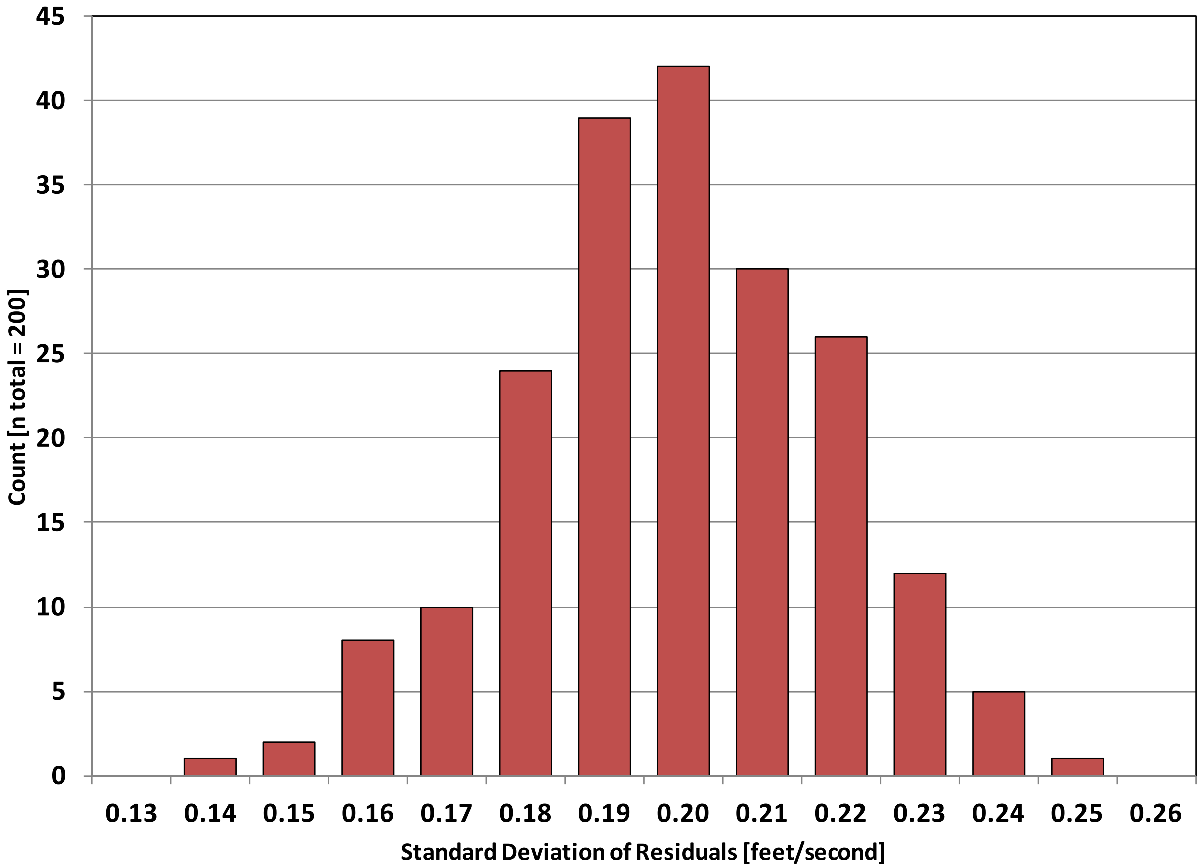

Figure 6: Distribution of the Standard Deviations of the Residuals for the 200 permutations of 20 randomly selected results.

In addition to evaluating the residuals of the 200 permutations, the Pearson's Correlation Coefficients (R^2) of each of the regression models were also analysed. The mean R^2 value was 0.991 with a range from 0.982 to 0.998. Figure 7 is a histogram of the Pearson Correlation Coefficients. In this analysis greater than 50% (114) of the 200 permutations yielded R^2 values higher than the full data set of 45 tests (R^2 = 0.991), indicating the strength of the model even with significantly fewer contributors.

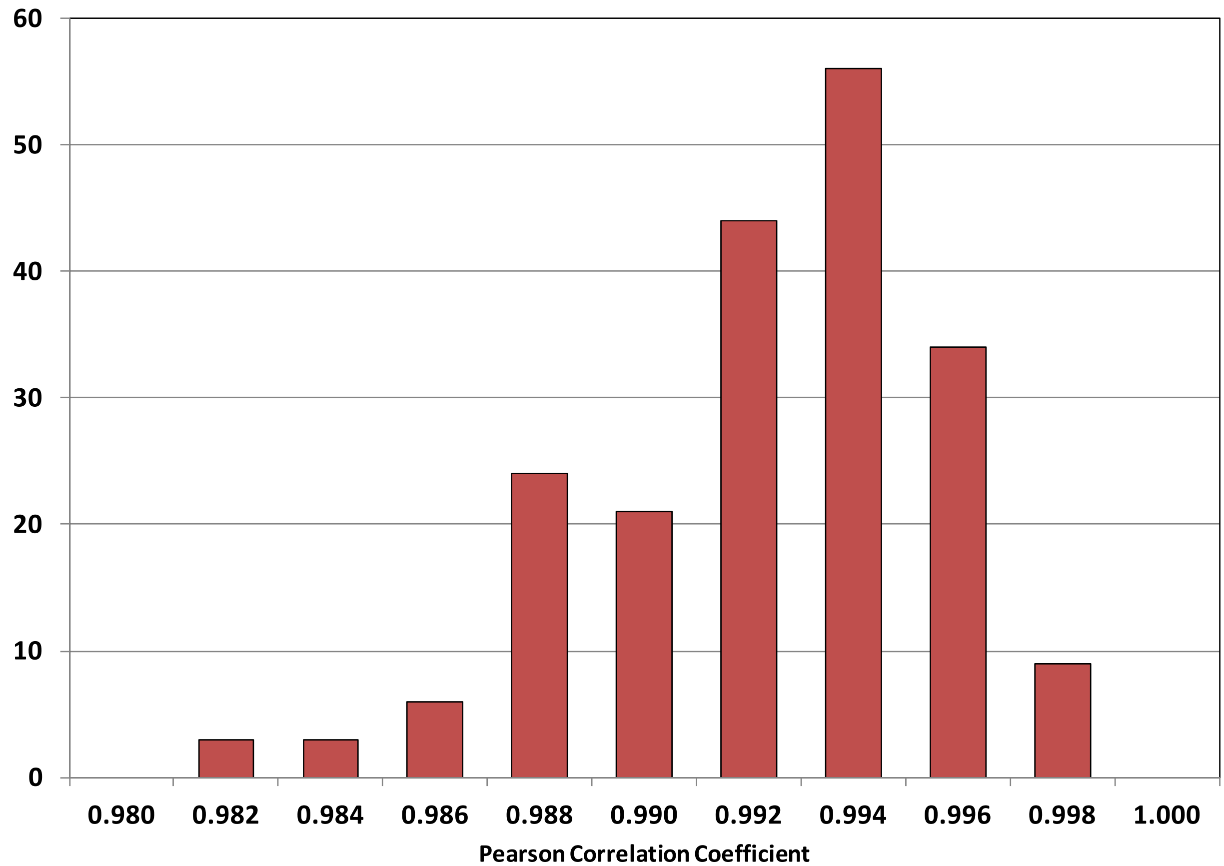

Figure 7: Distribution of Pearson Correlation Coefficients for the n = 200 permutations of 20 randomly selected results.

4. Conclusions

This study has demonstrated that the measure of Initial Velocity can be accurately predicted with the using normal rebound speed from COR testing (V_e) and normal contact time (t_c). The model developed is quite robust as a very broad population of balls was utilised in its development. The balls included in these tests had properties uniformly distributed over the full range of Initial Velocity and of contact time typically observed. Improvements to this model over simpler models were made by improving the quality of rebound velocity data through the inclusion of the actual rebound angle and measuring the contact time and COR simultaneously. The current model predicts Initial Velocity within ±0.39 ft/s at a 95% prediction interval. The bootstrapping analysis of this data verified the robustness of the model, as worst case scenarios of the standard deviation of residuals was 0.24 ft/s were found using as few as 20 sets of data.